International Civil Aviation Organization

The 17th Meeting of the Regional Airspace Safety Monitoring Advisory Group (RASMAG/17)

Bangkok, Thailand, 28 – 31 August 2012

Agenda Item 3: Reports from Asia/Pacific RMAs and EMAs

BOBASMA REPORT

(Presented by India/BOBASMA)

SUMMARY

This paper presents the 50NM Reduced Horizontal Separation Post Implementation Safety Assessment conducted by BOBASMA. The Safety Assessment was conducted using the Traffic Sample Data of Chennai, Mumbai and Kolkata for the month of June 2012. The Report also contains a summary of large lateral deviations and large longitudinal errors received by BOBASMA for the period 01 July 2011 to 30 June 2012.

This paper relates to –

Strategic Objectives:

*A: **Safety** – Enhance global civil aviation safety*

*C: **Environmental Protection and Sustainable Development of Air Transport** – Foster harmonized and economically viable development of international civil aviation that does not unduly harm the environment*

Global Plan Initiatives:

GPI-8 Collaborative airspace design and management
GPI-16 Decision support systems and alerting systems
GPI-21 Navigation Systems
GPI-22 Communication infrastructure

1. INTRODUCTION

1.1 The Bay Of Bengal Arabian Sea Indian Ocean Safety Monitoring Agency (BOBASMA), has been entrusted with the responsibility to provide safety monitoring services for the implementation of 50 NM reduced horizontal separation in the Bay of Bengal Arabian Sea India Ocean (BOBASIO) airspace.

1.2 50 NM Reduced Horizontal Separation was successfully implemented on Phase 1 routes, N571 & P762 from 30th June 2011 and on Phase 2 routes L301, L507, L509, L510, L759, M300, M770, N563, N877, N895, P570, P574, P628 & P6462 starting from 15th December 2011 based on the "Implementation readiness plan" formulated by the BOB-RHS/TF/6 in September 2011.

1.3 This paper presents the post-implementation safety assessment conducted by BOBASMA using the Traffic Sample Data of Chennai, Mumbai and Kolkata for the month of June 2012. The report also contains a listing of the Large Lateral Deviation and Large Longitudinal Error reports received by BOBASMA for the period 1st July 2011 to 30th June 2012 and a discussion on the various monitoring activities conducted by BOBASMA for the introduction of 50 NM RHS.

1.4 The objective of the safety assessment is to confirm that post implementation of 50 NM RHS in the BOBASIO airspace, the regionally established target level of safety is met for the continued safe use of 50 NM reduced longitudinal separation.

2. DISCUSSION

2.1 BOBASMA conducted the pre-implementation Safety Assessment for the introduction of 50 NM RHS on Phase 1 and 2 routes. **Table 1** shows the list of Safety Assessments conducted by BOBASMA till date, for the introduction and continued safe use of 50 NM RHS on RNP10 routes within BOBASIO airspace.

S. No	**Safety Assessment**	**Date**	**Lateral Collision Risk**	**Longitudinal Collision Risk**	**TLS**
1.	Phase1 Pre-implementation Safety Assessment	Feb 2011	0.601881×10^{-9}	0.371804×10^{-9}	5×10^{-9}
2.	Revised Safety Assessment	Aug 2011	0.621694×10^{-9}	0.964658×10^{-9}	5×10^{-9}
3.	Phase2 Pre-implementation Safety Assessment	Nov 2011	0.760904×10^{-9}	3.1759×10^{-9}	5×10^{-9}
4.	Periodic Safety Assessment	Feb 2012	1.04405×10^{-9}	0.67326×10^{-9}	5×10^{-9}

Table1: Safety assessments conducted by BOBASMA

Data Collection

2.2 For the purpose of this safety assessment the most recent one month traffic sample for June 2012 was collected from the three Indian FIRs of Chennai, Mumbai and Kolkata. All the Oceanic RNP10 routes on which 50 NM RHS is implemented traverse through these three Indian FIRs, and as such provide the most representative data sample for statistical collision risk modeling.

2.3 The traffic sample data of Mumbai and Chennai were collected from the ATM automation systems and for Kolkata, the TSD was collated from controller Flight Progress Strips and the standalone FDPS & ADS/CPDLC sub systems. The data sample was collected as per the format prescribed in the EMA Hand book. The collected data was processed for any errors in collection and where errors were detected, the concerned ATS unit was asked to forward all the relevant records and the errors checked and corrected.

Gross Navigational Errors (LLD & LLE)

2.4 The previous 12 month data on Large Lateral Deviations and Large Longitudinal Error received by BOBASMA from the ACCs/OCCs participating in the formal monitoring program for horizontal deviations in the BOBASIO airspace has been used for calculation of the collision risk estimates.

Application of the 50NM Separation Standard

2.5 Data on the number of aircraft pairs provided with 50 NM reduced longitudinal separation was collected from Mumbai, Chennai and Kolkata from the first day of implementation in the respective FIR. The data on the observed application of reduced horizontal separation is presented month wise from July 2011 to June 2012 for Chennai and Mumbai and from December 2011 to June 2012 for Kolkata.

2.6 **Figure 1** shows the percentage of aircraft operations with 50 NM reduced longitudinal separation observed in Chennai FIR where 50 NM reduced longitudinal separation is applied between suitably equipped aircraft along nine RNP10 routes. **Figure 2** shows the percentage of aircraft operation with 50 NM RLS observed in Mumbai FIR along the six RNP10 routes on which it has been implemented. **Figure 3** contains the observed percentage of operations with 50 NM RHS on the nine RNP10 routes within Kolkata FIR.

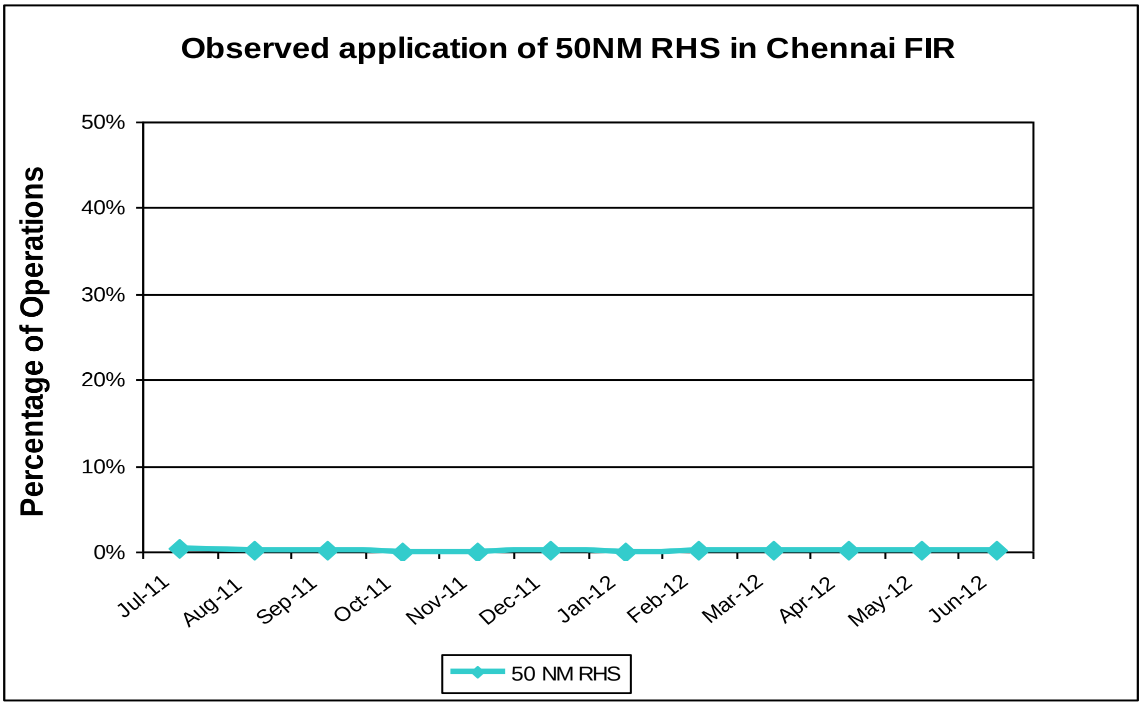

Figure 1: Percentage of operations with observed 50 NM RHS in Chennai FIR

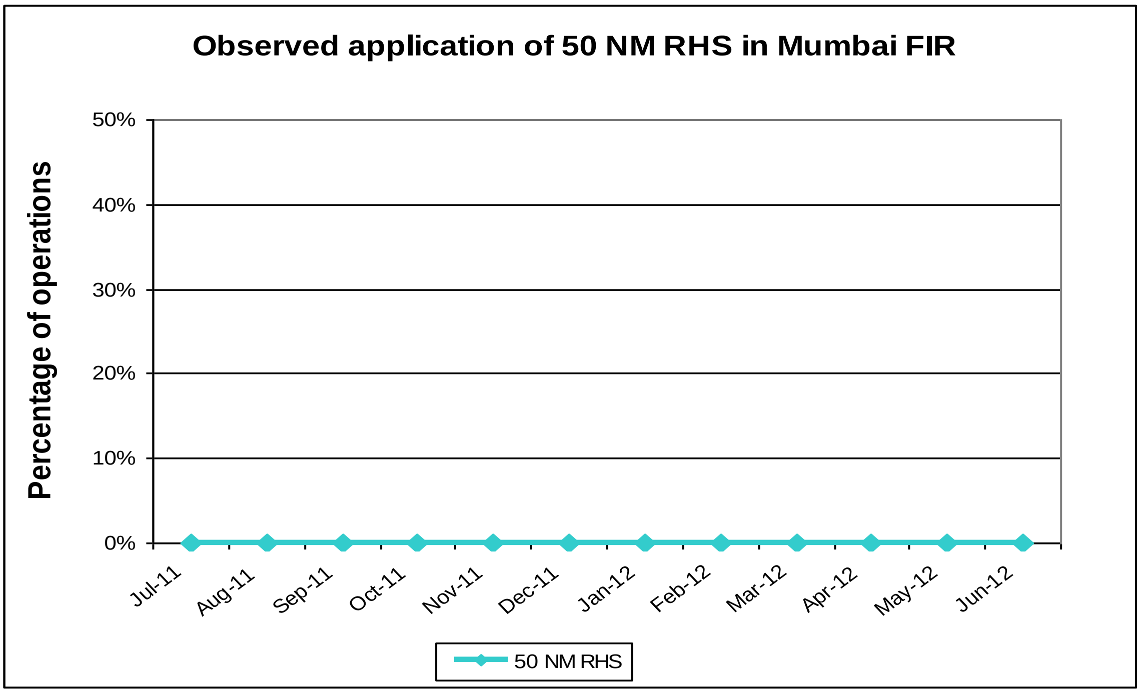

Figure 2: Percentage of operations with observed 50 NM RHS in Mumbai FIR

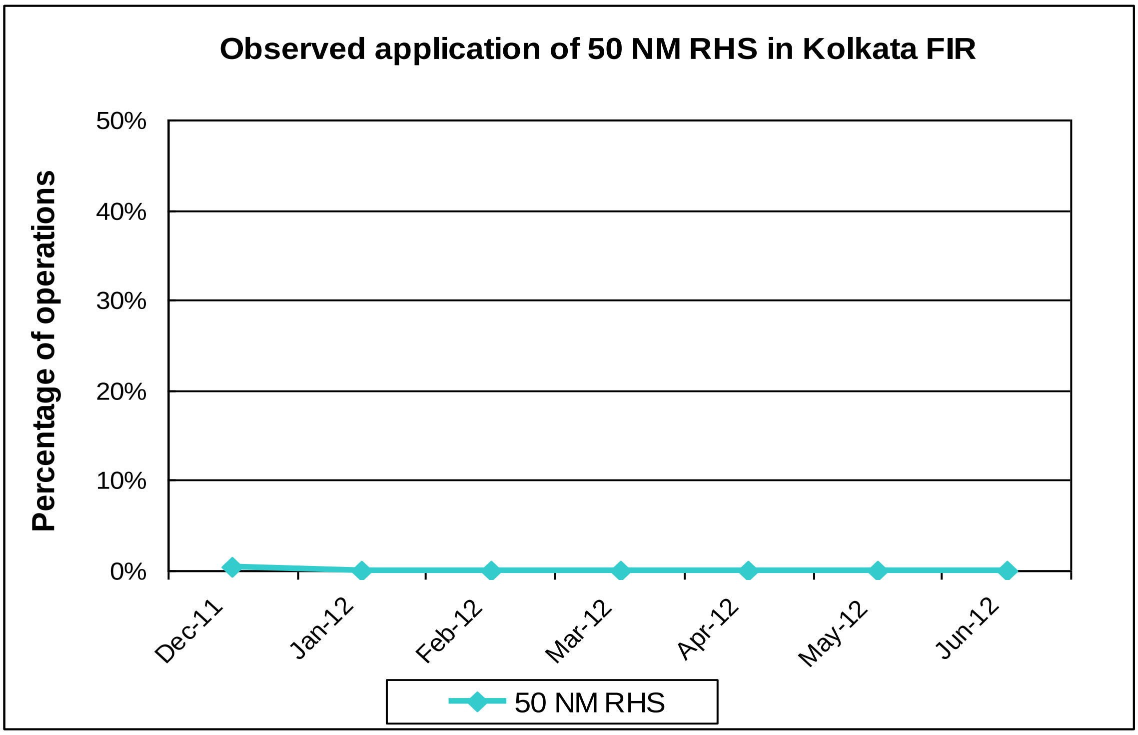

Figure 3: Percentage of operations with observed 50 NM RHS in Kolkata FIR

2.7 The average percentage of aircraft movements with observed application of 50 NM reduced longitudinal separation within Chennai, Mumbai and Kolkata is 0.149%, 0.024% and 0.069% respectively.

Horizontal Collision Risk Estimate for BOBASIO Airspace

2.8 The Lateral collision risk is estimated to be 0.740919×10^{-9} and the longitudinal collision risk 1.05943×10^{-9}, both of which are below the TLS of 5×10^{-9}. Thus it can be concluded that the post- implementation Safety Assessment supports the continued safe use of 50NM RLS on Phase-1 & 2 routes. The lateral and longitudinal collision risk assessment is given in Appendix-A.

Completion of Mentoring Period and Peer Review by SEASMA and AAMA

2.9 APANPIRG/22 meeting endorsed the establishment of EMA in India vide decision D 22/14.During the 15th RASMAG meeting, India agreed to enter into a 12 month period of mentoring by peer monitoring agencies SEASMA and AAMA. The 12 month mentoring period (September 2011 to August 2012) has been successfully completed by India. India would like to place it on record our sincere thanks to both SEASMA and AAMA for the mentorship provided by them.

3. ACTION BY THE MEETING

3.1 The meeting is invited to:

a) note the estimated lateral and longitudinal collision risks, both of which are well within the TLS of 5 x 10-9 and thus support the continued use of 50 NM RLS on Phase1 & 2 routes; and

b) discuss any relevant matters as appropriate.

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Appendix-A

Post-RHS Implementation Lateral and Longitudinal Collision Risk Assessment of the BOBASIO Region based on June 2012 TSD

1 Introduction

In this article we reinvestigate the collision risk between two aircraft flying over the Bay of Bengal, Arabian Sea, and Indian Ocean (BOBASIO) region. This safety assessment is undertaken jointly by the *Airports Authority of India (AAI)* and the *Indian Statistical Institute*, Delhi Centre under the MoA signed between the two organizations on January 13, 2011. The goal of this study is to confirm that the Target Level of Safety (TLS) which is 5 x 10^{-9} fatal accidents per flight hour, is currently met.

Note that in the following routes in the BOBASIO region reduced horizontal separation (RHS) was introduced in two phases:

- Phase 1: On the routes N571 and P762 from July 2011 and
- Phase 2: On the routes L301, L507, L509, L510, L759, M300, M770, N563, N877, N895, P570, P574, P628 and P646 from December 15, 2011.

Thus currently the separation standards are as follows

- for lateral separation it is at least 50 NM between all the parallel routes;
- for longitudinal it has been reduced to 50 NM between front and behind aircrafts.

In this article we carry out the quantitative risk analysis based on two types of datasets supplied by three Indian FIRs.

- **Traffic Sample Data (TSD):**

Traffic sample data from Chennai, Kolkata, and Mumbai FIRs for the month of June 2012 was used. The original data contained several anomalies, which we tried to detect and remove. Briefly, the following initial filtering criteria were used:

- Duplicate records were removed.
- Records with Exit time less than Entry time were removed.
- Records with flight level less than F280 were removed.

22872 records that were retained after filtering were considered for the subsequent statistical analysis. Figure 1 provides a graphical summary of the number of flights by route and flight level for RHS routes.

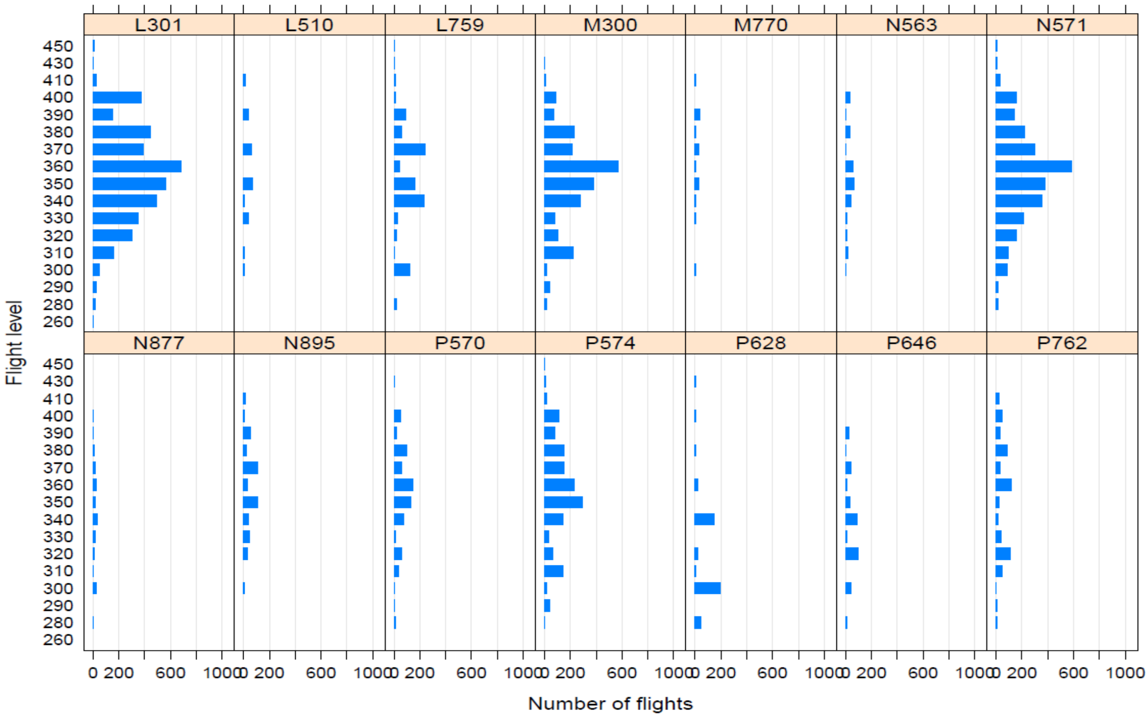

Figure 1: Number of flights by route and flight level in Chennai, Kolkata, and Mumbai FIRs, based on June 2012 TSD.

- **Gross Navigational Error (GNE) Data:**

Reports of Gross Navigational Errors for the preceding twelve month period were received from Chennai, Kolkata and Mumbai FIRs, as summarized in Table 1.

Year	MONTH	FIR	Flights	LLE	LLD
2011	JULY	KOLKATA	1919	0.00	0.00
2011	AUGUST	KOLKATA	1868	0.00	0.00
2011	SEPTEMBER	KOLKATA	1765	0.00	0.00
2011	OCTOBER	KOLKATA	2082	0.00	0.00
2011	NOVEMBER	KOLKATA	2113	0.00	0.00
2011	DECEMBER	KOLKATA	2318	0.00	0.00
2012	JANUARY	KOLKATA	2462	0.00	0.00
2012	FEBRUARY	KOLKATA	2251	0.00	0.00
2012	MARCH	KOLKATA	2259	0.00	0.00
2012	APRIL	KOLKATA	1999	0.00	0.00
2012	MAY	KOLKATA	1986	0.00	0.00
2012	JUNE	KOLKATA	1790	0.00	0.00
2011	JULY	CHENNAI	1963	0.00	0.00
2011	AUGUST	CHENNAI	2019	0.00	0.00

2011	SEPTEMBER	CHENNAI	1872	0.00	0.00
2011	OCTOBER	CHENNAI	1957	0.00	0.00
2011	NOVEMBER	CHENNAI	1848	0.00	0.00
2011	DECEMBER	CHENNAI	5026	0.00	0.00
2012	JANUARY	CHENNAI	9623	0.00	0.00
2012	FEBRUARY	CHENNAI	9058	0.00	0.00
2012	MARCH	CHENNAI	9711	0.00	0.00
2012	APRIL	CHENNAI	9992	0.00	0.00
2012	MAY	CHENNAI	10250	0.00	0.00
2012	JUNE	CHENNAI	7478	0.00	0.00
2011	JULY	MUMBAI	8283	0.00	0.00
2011	AUGUST	MUMBAI	11019	0.00	0.00
2011	SEPTEMBER	MUMBAI	11616	0.00	0.00
2011	OCTOBER	MUMBAI	12659	0.00	0.00
2011	NOVEMBER	MUMBAI	12869	0.00	0.00
2011	DECEMBER	MUMBAI	12276	0.00	0.00
2012	JANUARY	MUMBAI	12290	0.00	0.00
2012	FEBRUARY	MUMBAI	11531	0.00	0.00
2012	MARCH	MUMBAI	12077	0.00	0.00
2012	APRIL	MUMBAI	11475	0.00	0.00
2012	MAY	MUMBAI	11728	0.00	0.00
2012	JUNE	MUMBAI	10472	0.00	0.00
2011	JULY	MANGALORE	4089	0.00	0.00
2011	AUGUST	MANGALORE	3983	0.00	0.00
2011	SEPTEMBER	MANGALORE	3786	0.00	0.00
2011	OCTOBER	MANGALORE	3584	0.00	0.00
2011	NOVEMBER	MANGALORE	3309	0.00	0.00
2011	DECEMBER	MANGALORE	3644	0.00	0.00
		Total	256299	0.00	0.00

Table 1: Summary of reports of Gross Navigational Errors.

In Section 2 we discuss the risk assessment for the lateral direction and Section 3 gives the same for the longitudinal direction.

2 Lateral Collision Risk Assessment

2.1 Lateral Collision Risk Model

In order to compute the level of safety for lateral deviations of operations on the BOBASIO region we use the Reich Lateral Collision Risk Model. It models the lateral collision risk due to the loss of lateral separation between aircraft on adjacent parallel tracks flying at the same flight level. The model is as follows:

$$N_{ay} = P_y(S_y)\,P_z(0)\,\frac{\lambda_x}{S_x}\left\{E_y(\text{same})\left[\frac{\overline{|\Delta V|}}{2\lambda_x}+\frac{\overline{|\dot{y}(S_y)|}}{2\lambda_y}+\frac{\overline{|\dot{z}|}}{2\lambda_z}\right]+E_y(\text{opp})\left[\frac{2\overline{|V|}}{2\lambda_x}+\frac{\overline{|\dot{y}(S_y)|}}{2\lambda_y}+\frac{\overline{|\dot{z}|}}{2\lambda_z}\right]\right\}. \quad (1)$$

We would like to note that same model has been used for the safety assessment study of the South China Sea which was carried out by SEASMA and also in European safety assessment which was carried out for EUR/SAM corridor.

The parameters in the equation (1) are defined as follows:

- N_{ay} := Expected number of fatal accidents (two for every collision) per flight hour due to the loss of lateral separation between co-altitude aircrafts flying on tracks with planned S_y NM lateral separation.
- S_y: = Minimum planned lateral separation.
- λ_x := Average length of an aircraft flying in BOBASIO region.
- λ_y := Average wingspan of an aircraft flying on BOBASIO region.
- λz := Average height of an aircraft flying on BOBASIO region.
- $P_y(S_y)$:= The probability of lateral overlap of aircraft nominally flying on adjacent flight paths, separated by Sy.
- $P_z(0)$:= Probability of vertical overlap of aircraft assigned at the same flight path at the same flight level.
- S_x: = Length of half the interval in NM used to count proximate aircraft at adjacent routes.
- E_y(same) := Same direction lateral occupancy at same assigned flight level.
- E_y(opp) := Opposite direction lateral occupancy at same assigned flight level.
- $\overline{|\Delta V|}$:= Average relative speed of two aircraft flying on parallel routes in same direction.
- $\overline{|V|}$:= Average ground speed on an aircraft.
- $|\dot{Y}(S_y)|$:= Average relative lateral speed of aircraft pair at loss of planned lateral separation of S_y.
- $\overline{|\dot{z}|}$:= Average relative vertical speed of a co-altitude aircraft pair assigned to the same route.

A collision, and consequently two fatal accidents, can only occur if there is overlap between two aircraft in all three dimensions simultaneously. Equation (1) gathers the product of the probabilities of losing separation in each one of the three dimensions.

As it has already been said, P_z (0) is the probability of vertical overlap; P_y (S_y) is the probability of lateral overlap and the combinations of $\frac{\lambda_x}{S_x}E_y\,(\mathrm{same})$ and $\frac{\lambda_x}{S_x}E_y\,(\mathrm{opp})$ relate to the probability of longitudinal overlap of aircraft on adjacent parallel tracks and at the same flight level. All the probabilities can be interpreted as proportions of flight time in the airspace during which overlap in the pertinent dimension occurs. As the collision risk is expressed as the expected number of fatal accidents per flight hour, the joint overlap probability must be converted into number of events involving joint overlap in the three dimensions, relating overlap probability with passing frequency. Here we note that passing frequency between two adjacent routes is the average number of events, per flight hour, in which two aircraft are in longitudinal overlap when travelling in the opposite or same direction at the same flight level. This is achieved by means of the expressions within square brackets in Equation (1). Each of the terms within square brackets represents the reciprocal of the average duration of an overlap in one of the dimensions. For example, $\frac{\overline{|\Delta V|}}{2\lambda_x}$ is the reciprocal of the average duration of an overlap in the longitudinal direction for same direction traffic. In the case of longitudinal direction too, but for opposite direction, the average relative speed is $2\overline{V}$ and the average overlap time $\frac{\overline{V}}{\lambda_x}$ The model is based on the following hypothesis:

- All routes are parallel. [1]
- All collisions normally occur between aircraft on adjacent routes, although, if the probability of overlap is significantly large, they may also occur on non-adjacent routes.
- The entry times into the track system are statistically independent.
- The lateral deviations of aircraft on adjacent tracks are statistically independent.
- The vertical, longitudinal and lateral deviations of an aircraft are statistically independent.
- The aircraft are replaced by rectangular boxes.
- There is no corrective action by pilots or ATC when two aircraft are about to collide.

The model also assumes that the nature of the events making up the lateral collision risk is completely random. This implies that any location within the system can be used to collect a representative data sample on the performance of the system.

2.2 Estimated Values of the Parameters and Estimated Lateral Collision Risk

The following table gives the values of the parameters of the right-hand side of the equation (1) which are obtained from our analysis.

Parameter	Estimated Values	Source of the Estimate
S_y	50 NM	Current minimum lateral separation.
λ_x	0.03124121 NM	Estimated from TSD (see Section 2.3).
λ_y	0.0289324 NM	Estimated from TSD (see Section 2.3).
λ_z	0.008834732 NM	Estimated from TSD (see Section 2.3).
P_y (50)	2.59137×10^{-8}	Estimated using a mixture model (see Section 2.4).
P_z (0)	0.538	Conservative value used in previous safety assessments (see Section 2.5).
S_x	50 NM	Reduced longitudinal separation.
E_y(same)	0.05953902	Estimated from the TSD (see Section 2.6).
E_y(opp)	0	No opposite directional lateral occupancy at same assigned flight level.
$\overline{\lvert\Delta V\rvert}$	20 knots	Value obtained from TSD (see Section 2.7).
$\overline{\lvert\dot{y}(50)\rvert}$	75 knots	Conservative value taken from EMA Handbook (see Section 2.8).
$\overline{\lvert\dot{z}\rvert}$	1.5 knots	Conservative value as per EMA Handbook (see Section 2.9).

Finally this leads to the following estimate for the lateral collision risk N_{ay}.

$$\mathbf{N}_{ay} = 0.740919 \times 10^{-9}$$

[1] In the BOBASIO region there are cross routes, such as, P762. A stricter time separation is imposed on such a route but we ignore that to be more conservative about our estimat

2.3 Estimating Average Aircraft Dimensions

Table 2 summarizes the distribution of aircraft population in the TSD. To be conservative, we used the maximum aircraft dimensions.

Type	Length	Wingspan	Height	Flights
B77W	73.90	64.80	18.50	3207
B738	39.20	34.40	12.57	2201
A320	37.57	34.10	11.76	2110
A332	58.80	60.30	17.40	1823
B772	63.70	60.90	18.40	1410
A333	63.60	60.30	16.85	1232
B744	70.60	64.80	19.40	1122
A388	73.00	79.80	24.10	808
A321	44.51	34.10	11.76	668
A343	63.60	60.30	16.85	471
B773	73.90	60.90	18.40	418
A319	33.84	34.10	11.76	391
B77L	63.70	64.80	18.30	325
MD11	61.20	51.70	17.60	167
A306	54.10	44.84	16.54	167
B763	54.90	47.60	15.90	166
A310	46.66	43.90	15.80	134
A346	75.30	63.45	17.30	88
B737	33.60	34.30	12.60	50
GLEX	30.30	26.90	7.60	20
B74S	56.30	59.60	20.00	20
A345	67.90	63.45	17.10	17
GLF4	26.90	23.70	7.40	16
B741	70.60	59.60	19.30	16
B752	47.30	38.10	13.60	14
GLF5	29.40	28.50	7.50	12
CL60	20.85	19.60	6.30	12
B742	70.60	59.60	19.30	12
B743	70.60	59.60	19.30	10
F900	20.20	19.30	7.60	6
H25B	15.60	15.70	5.40	5
F2TH	20.20	19.30	7.10	5

Table 2: Dimensions of aircraft types, along with number of records in the TSD

2.4 Estimating Probability of Lateral Overlap: $P_y(S_y)$

The probability of lateral overlap of aircraft nominally flying on adjacent flight paths, separated by S_y, is denoted by $P_y(S_y)$ and is defined as

$$P_y(S_y) := \mathbf{P}(|S_y + Y_1 - Y_2| \le \lambda_y), \quad (2)$$

where Y1 and Y2 are assumed to be the lateral deviations of two aircraft which are nominally separated by S_y. We assume that Y1 and Y2 are identically distributed but statistically independent with a distribution F_y.

We model F_y as mixture distribution having a *core* distribution G_y and a *non-core* distribution H_y.

- The *core* distribution G_y, represents errors that derive from standard navigation system deviations. These errors are always present, as navigation systems are not perfect and they have a certain precision.
- The *non-core* distribution H_y, represents *Gross Navigation Errors (GNE),* that corresponds to what may be viewed as non-nominal performance.

We assume that a standard navigation system error represented by the core distribution may take large values but the non-core distribution representing gross navigation errors can only take large values. But in most cases it is impossible to determine with certainty if a given observed lateral error arose from the core or from the tail term of the distribution. Therefore, the overall lateral deviation distribution is modeled as:

$$F_y(y) = (1 - \alpha)G_y(y) + \alpha H_y(y). \quad (3)$$

The mixing parameter α is the probability of a *gross navigational error*.

The core lateral deviation distribution G_y is modeled by a Double Exponential distribution with a parameter $\beta_y > 0$ as the rate, that is, if $Y_1 \sim G_y$ then

$$\mathbf{P}(|Y_1| > y) = e^{-\beta_y y},$$

where $y \ge 0$. In other words we assume that the core distribution has a density of the form

$$g_y(y) = \frac{\beta_y}{2} e^{-\beta_y |y|}.$$

Finally the non-core distribution H_y is modeled by a "Separated Double Exponential" distribution with parameters $\mu_y > 0$, representing the "separation and $\gamma_y > 0$ the rate parameter, that is, if $Y_2 \sim H_y$ then

$$\mathbf{P}(Y_2 > \mu_y + y) = \frac{1}{2} e^{-\gamma_y y} \text{ and}$$

$$\mathbf{P}(Y_2 < -\mu_y - y) = \frac{1}{2} e^{\gamma_y y},$$

where $y \ge 0$. This really means that the non-core distribution H_y gives no mass in $[-\mu_y, \mu_y]$ and outside it decays as a Double Exponential distribution with rate parameter γ_y.

The density of this distribution is given by

$$h_y(y) = \begin{cases} \frac{\gamma_y}{2} e^{\gamma_y (y+\mu_y)} & \text{if } y < -\mu_y \\ 0 & \text{if } -\mu_y \le y \le \mu_y \\ \frac{\gamma_y}{2} e^{-\gamma_y (y-\mu_y)} & \text{if } y > \mu_y \end{cases}.$$

This modeling is similar to what has been used by FAA and also in EUR/SAM except here we take a double exponential distribution, namely the core distribution to explain all the typical and atypical errors which are not a gross navigational error, and use the separated double exponential distribution for the gross navigational errors.

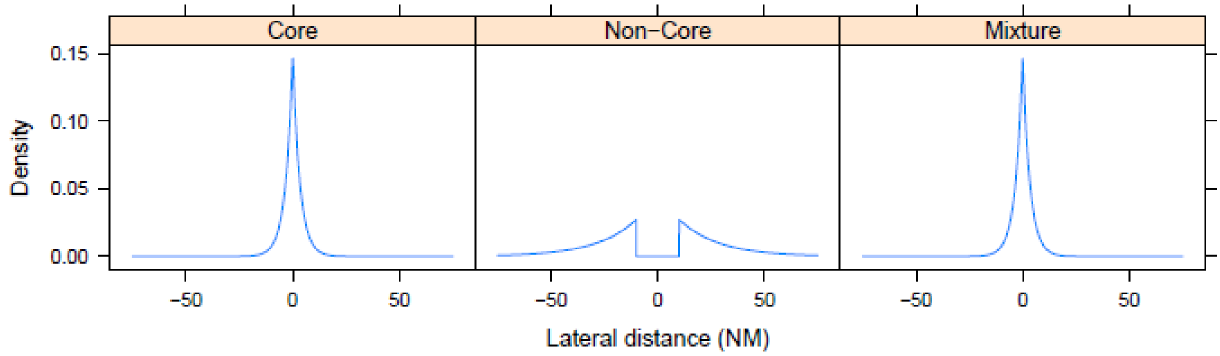

Figure 2: Modeling of lateral deviation.

This in turn gives a better understanding of the mixing parameter α which we estimate by taking the 95% upper confidence limit from the available GNE data. The formula comes out to be

$$\hat{\alpha} = 1 - (0.05)^{1/N} = 1.16884 \times 10^{-5},$$

where N = 256299 is the number of flights observed and no gross navigational errors were detected. More GNE data with no detected gross navigational error will increase the value of N and hence decrease the value of α which will lead to decrease in the risk.

Here we would like to note that even though the non-core distribution H_y has a discontinuous density h_y, it does not create difficulty in this risk assessment.

The parameter β_y is estimated under the RNP10 assumption of ±10 NM deviation with 95% confidence, this leads to the estimate

$$\hat{\beta}_y = -\frac{\log 0.05}{10} = 0.299573227.$$

The parameter μ_y is taken to be 10 based on RNP10 consideration and γ_y is then estimated by maximizing the wingspan overlap probability with S_y=50NM initial separation (see Figure 3). This is a conservative method similar to what has been used by FAA and also in EUR/SAM. The estimated value of γ_y is 0.05489708 leading to the estimated value of P_y (50) as 2.59137 x 10^{-8}.

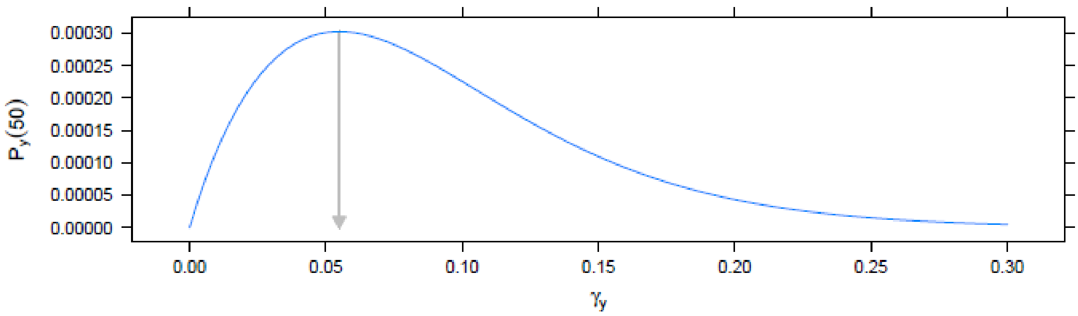

Figure 3: Wingspan overlap probability as a function of γ_y with S_y = 50 NM initial separation.

To be conservative, we also considered the possibility of unreported GNEs, and computed the estimates of P_y (50) and N_{ay} had we observed 1, 2, or 3 GNEs. The results, given below, are still well below the TLS.

No. of GNEs	P_y (50)	N_{ay}
0	2.60276×10^{-8}	0.930222×10^{-9}
1	2.82216×10^{-8}	1.00863×10^{-9}
2	3.04155×10^{-8}	1.08704×10^{-9}
3	3.26094×10^{-8}	1.16545×10^{-9}
4	3.48033×10^{-8}	1.24386×10^{-9}
5	3.69972×10^{-8}	1.32227×10^{-9}

The estimate of α does not have a nice formula when one or more GNEs are observed, but can be computed using numerical methods.

2.5 Estimating Probability of Vertical Overlap: $P_z(0)$

The probability of vertical overlap of aircraft nominally flying at the same flight level on laterally adjacent flight paths is denoted by $P_z(0)$. It is defined by

$$P_z(0) = \mathbf{P}(|Z_1 - Z_2| \leq \lambda_z) ,$$

where Z_1 and Z_2 are the height deviations of two aircraft nominally flying at the same flight levels on laterally adjacent flight paths.

We assume that Z_1 and Z_2 are statistically independent with distribution F_z. Unlike in the computation of $P_y(S_y)$ where we assume the lateral deviations follow a mixture distribution here we may assume that F_z is a Double Exponential distribution with parameter $\beta_z > 0$, that is, with density function

$$f_z(z) = \frac{\beta_z}{2} e^{-\beta_z |z|} .$$

One can then estimate $\beta_z > 0$ by

$$\widehat{\beta_z} = -\frac{\log 0.05}{0.032915} = 91.0142 .$$

This is under assumption that a typical aircraft stays within ±200 ft = ±0.032915 NM of its assigned flight level 95% of the time. This leads to an estimated value 0.3552838 for P_z (0). Unfortunately this analysis ignores both the effect of large height deviations (LHDs) and aircraft altimetry system errors (ASE) which are not estimable directly. So we use a conservative value of 0.538, as used by MAAR for vertical safety assessment in BOB region.

2.6 Estimating the Lateral Occupancy Parameters: E_y(same) and E_y(opp)

In equation (1) there are two occupancy terms, one for same direction occupancy E_y(same) and another one for opposite direction occupancy E_y(opp).

Same direction occupancy is defined as the average number of aircraft that are, in relation to a typical aircraft

- flying in the same direction as it;
- nominally flying on tracks one lateral separation standard away;
- nominally at the same flight level as it; and
- within a longitudinal segment centered on it.

The length of the longitudinal segment, $2S_x$, is usually considered to be the length equivalent to 20 minutes of flight resulting to a value of 160 NM. It has been verified that the relationship between S_x and the occupancy is quite linear.

A similar set of criteria can be used to define opposite direction occupancy, just replacing “flying in the same direction” by “flying in the opposite direction”. Occupancy, in general, relates to the longitudinal overlap probability and can be obtained by the equation

$$E_y = \frac{2T_y}{H},$$

where T_y represents the total proximity time generated in the system and H is the total flight hours generated in the system during the considered period of time.

We estimate this quantity by direct estimation from time at waypoint passing using the TSD. For this we compute the number of proximate pairs by comparing the time at which an aircraft on one route passes a waypoint with the time at which another aircraft on a

WP1	WP2	Proximate	Total
BIDEX	ORARA	0	560
IGOGU	IGREX	106	2504
NOPEK	IGOGU	170	2654
GIRNA	IDASO	176	2632
VATLA	ORARA	8	796
IGOGU	EMRAN	0	1882
LIBDI	MEPEL	0	78
RINDA	SAGOD	48	2244
MEPEL	IBITA	0	100
IBITA	TEBOV	0	22
SAGOD	IBITA	0	226
POMAN	IGAMA	218	3332
KITAL	LOTAV	34	822
OPIRA	IGAMA	0	2506
LOTAV	REXOD	22	800
TOTOX	REXOD	76	1586
TOTOX	PARAR	206	3052
ADPOP	SUGID	0	2932
RASKI	PARAR	622	4930
NOBAT	SUGID	974	7458
POMAN	ODOLI	0	826
KITAL	ASPUX	0	194
SULTO	DUGOS	0	334
UDULO	KAKIB	40	1742

BUBKO	DOPID	0	44
RIBRO	ELBAB	2	472
VATLA	MABUR	0	654

Table 3: Number of laterally proximate flights per route pair, based on TSD.

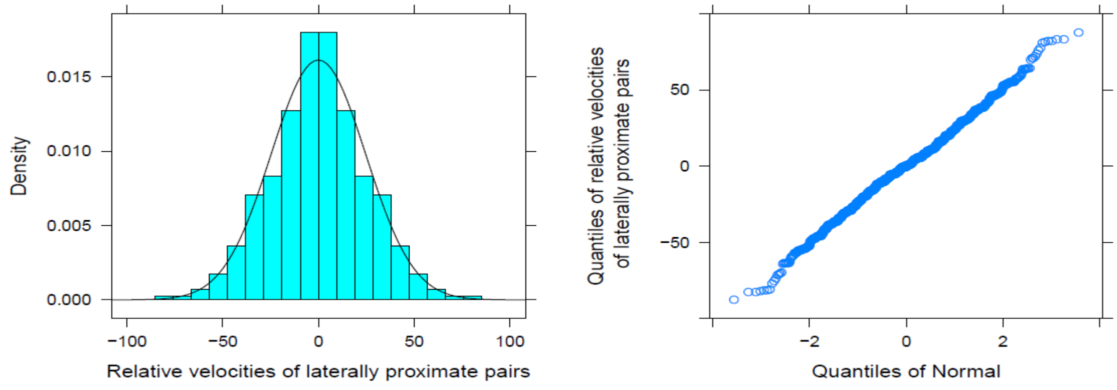

Figure 4: Distribution of relative velocities of laterally proximate pairs. The Normal distribution with sample standard deviation looks like a reasonable fit.

parallel route passes the homologous waypoint. When the difference between passing times is less than certain value, 10 minutes in this case, it is considered that there is a proximate pair in that pair of routes. Occupancy is then calculated using the following expression:

$$E_y = \frac{2n_y}{n},$$

where the numerator n_y is the number of proximate pairs and the denominator, n, is the the total number of aircraft. The observed number of proximate pairs and the total number of flights per route pair are summarized in Table 3.

2.7 Estimate of Average Relative Longitudinal Speed: $\overline{|\Delta V|}$

$\overline{|\Delta V|}$ is the average relative longitudinal speed between aircraft flying in the same direction. We estimate it from the TSD by taking the differences between the speeds of all the pairs of aircraft that constitute a lateral proximate pair in the same direction (see Figure 4). $\overline{|\Delta V|}$ is estimated as the mean absolute value of all the calculated differences, which turns out to be 19.21714. We use the conservative value 20. Here we note that the lateral proximate pairs are already determined while estimating the parameter E_y(same).

2.8 Estimate of Average Relative Lateral Speed: $\overline{|\dot{Y}(S_y)|}$

$\overline{|\dot{Y}(S_y)|}$ is the average relative lateral cross-track speed between aircraft, flying on adjacent routes separated by S_y NM at the same flight level, that have lost their lateral separation. The estimation of this parameter generally involves the extrapolation of radar data, speeds and lateral deviations, but such radar data were not available for this study. So we take a conservative value 75 knots as per the EMA Handbook.

2.9 Estimate of Average Relative Vertical Speed: $\overline{|\dot{z}|}$

$\overline{|\dot{z}|}$ denotes the average modulus of the relative vertical speed between a pair of aircraft on the same flight level of adjacent tracks that has lost lateral separation. It is generally assumed that $\overline{|\dot{z}|}$ is independent of the size of the lateral separation between the aircraft and, for aircraft in level flight, it can also be considered that there is no dependency of $\overline{|\dot{z}|}$ with the vertical separation between the aircraft. As noted by various agencies data on $\overline{|\dot{z}|}$ are relatively scarce but typically taken as 1.5 knots which is considered to be conservative (see EMA Handbook).

3 Longitudinal Collision Risk Assessment

In order to compute the level of safety for longitudinal deviations of operations on the BOBASIO region we use the Longitudinal Collision Risk Model. It models the longitudinal collision risk due to the loss of longitudinal separation between aircrafts flying on the same route at the same flight level. The model is as follows:

$$N_{ax} = P_y(0)\,P_z(0)\,\frac{2\lambda_x}{|\dot{x}|}\left(\frac{\overline{|\dot{x}|}}{2\lambda_x} + \frac{\overline{|\dot{y}(0)|}}{2\lambda_y} + \frac{\overline{|\dot{z}|}}{2\lambda_z}\right) \times \left[\sum_{k=m}^{M} 2Q(k)\,\mathbf{P}(K > k)\right]. \qquad (4)$$

We would like to note that the same model has been used for the safety assessment study of the South China Sea which was carried out by SEASMA.

The parameters in the equation (4) are defined as follows:

- N_{ax} := Expected number of fatal accidents (two for every collision) per flight hour due to the loss of longitudinal separation between co-altitude aircrafts flying on the same track with planned minimum *m* NM longitudinal separation.
- *m* := Minimum longitudinal separation in NM.
- M := Maximum initial longitudinal separation between aircraft pair which will be monitored by ATC in order to prevent loss of longitudinal separation standard.
- λ_x := Average length of an aircraft flying on BOBASIO region.
- λ_y := Average wingspan of an aircraft flying on BOBASIO region.
- λ_z := Average height of an aircraft flying on BOBASIO region.
- P_y (0) := Probability of lateral overlap of aircraft assigned same flight route and flight level.
- P_z (0) := Probability of vertical overlap of aircraft assigned at the same flight path at the same flight level.
- $\overline{|\dot{x}|}$:= Minimum relative along-track speed necessary for following aircraft in a pair separated by *m* NM at a reporting point to overtake lead aircraft at the next reporting point.
- $\overline{|\dot{Y}(0)|}$:= Relative across-track speed of same route aircraft pair.
- $\overline{|\dot{z}|}$:= Average relative vertical speed of a co-altitude aircraft pair assigned to the same route.
- Q (k):= Proportion of aircrafts for which the following aircraft has initial longitudinal separation k.
- $\mathbf{P}$ (K > k) := Probability that a pair of same route co-altitude aircraft with initial longitudinal separation k will lose at least as much as k longitudinal separation before correction by ATC.

Once again, a collision, and consequently two fatal accidents, can only occur if there is overlap between two aircraft in all three dimensions simultaneously. Equation (4) gathers the product of the probabilities of losing separation in each one of the three dimensions. The equation is derived under similar assumption as done in the case of lateral collision risk assessment.

We should note here the first part of the right-hand side of the equation (4) gives the probability of a collision given an event of overtake of a front aircraft by a behind aircraft when both are nominally flying at the same route at the same flight level, and the second part which is inside the square bracket is the expected number of aircrafts involved in such overtake events.

3.1 Estimated Values of the Parameters and Estimated Longitudinal Collision Risk

The following table gives the values of the parameters of the right-hand side of the equation (4) which are obtained from our analysis.

Parameter	Estimated Values	Source of the Estimate
m	50 NM	Current minimum longitudinal separation (due to RHS).
M	160 NM	Conservative value corresponding to 20 minutes separation.
λ_x	0.03124121 NM	Estimated from TSD (See Section 2.3).
λ_y	0.0289324 NM	Estimated from TSD (see Section 2.3).
λ_z	0.008834732 NM	Estimated from TSD (see Section 2.3).
$P_y(0)$	0.2	Conservative estimate (see Section 3.2).
$P_z(0)$	0.538	Conservative value used in previous safety assessments (see Section 2.5).
$\lvert\dot{x}\rvert$	19 knots	Conservative estimate using speed and distance between way points (see Section 3.3).
$\lvert\dot{y}(0)\rvert$	1 knot	RASMAG/9 safety assessment (see Section 3.4).
$\lvert\dot{z}\rvert$	1.5	Conservative value as per EMA Handbook (see Section 2.9).
$Q(k)$	See Table 4	Obtained from TSD (see Section 3.5).
$P(K > k)$	See Table 4	Computed using normal model on speed (see Section 3.6).

Finally this leads to the following estimate for the longitudinal collision risk N_{ax}.

$$N_{ax} = 1.05943 \times 10^{-9}$$

3.2 Estimating Probability of Lateral Overlap: $P_y(0)$

$P_y(0)$ is defined as the probability of lateral overlap of aircraft nominally flying at adjacent flight levels on same route. We can now use the same mixture model of Section 2.4 to compute this parameter by substituting $S_y = 0$ in the equation (2). This leads to an estimate of $P_y(0)$ as 0.2.

However as noted earlier in the EUR/SAM report, this factor $P_y(0)$ has a significant effect on the risk estimate. Therefore, it should not be underestimated. $P_y(0)$ will increase as the lateral navigational performance of typical aircraft improves, causing a corresponding increase in the collision risk estimate. As reported in the EUR/SAM report, the RGCSP was aware of this problem and attempted to account for improvements in navigation systems when defining the RVSM global system performance specification. Based on the performance of highly accurate area navigation systems observed in European airspace, which demonstrated lateral path-keeping errors with a standard deviation of 0.3 NM, the RGCSP adopted a value of 0.059 as the value of $P_y(0)$ for the global system performance.

As observed by many monitoring agencies and pointed out to us by AAMA the RGCSP value of $P_y(0)$ does not acknowledge the close track-keeping observed with RNP 4 or GNSS-equipped RNAV 10/RNP 10 aircraft. Further the EMA Handbook recommends to take a conservative value as 0:2. So we take this conservative value for our analysis as well.

k (mins)	k (NM)	Q(k)	P(K > k)
7	56	0.0004713487	1.29802 x 10^{-5}
8	64	0.0010773685	9.91472 x 10^{-7}
9	72	0.0011447041	8.07348 x 10^{-8}
10	80	0.0074742442	9.96907 x 10^{-9}
11	88	0.0077435863	1.70156 x 10^{-9}
12	96	0.0083496061	3.16917 x 10^{-10}
13	104	0.0080802640	5.97554 x 10^{-11}
14	112	0.0080802640	1.12797 x 10^{-11}
15	120	0.0095616457	2.12937 x 10^{-12}
16	128	0.0092923035	4.01982 x 10^{-13}
17	136	0.0110430274	7.58859 x 10^{-14}
18	144	0.0084842772	1.43257 x 10^{-14}
19	152	0.0093596391	2.70439 x 10^{-15}
20	160	0.0086862838	5.10533 x 10^{-16}

Table 4: Estimated values of Q(k) and **P** (K > k)

3.3 Estimation of the Parameter $\overline{|\dot{x}|}$

$\overline{|\dot{x}|}$ is defined as the minimum relative along-track speed necessary for following aircraft in a pair separated by *m* NM at a reporting point to overtake lead aircraft at the next reporting point. Thus if d is the distance between the two way points and v_0 is the speed of the front aircraft then $\overline{|\dot{x}|}$ can be computed by the equation.

$$\frac{d-m}{v_0} = \frac{d}{v_0 + \overline{|\dot{x}|}},$$

leading to

$$\overline{|\dot{x}|} = \frac{m v_0}{d-m}.$$

We conservatively estimate it by taking v_0 as the the minimum speed observed in TSD which is 360 NM per hours and the maximum distance between two waypoints on the routes which we study which is d = 971 NM. With *m* = 50 NM the final estimate turns out to be $\overline{|\dot{x}|}$ = 19.5439739413681 knots. We use a conservative value of 19 knots.

3.4 Estimation of the Parameter: $\overline{|\dot{y}(0)|}$

$\overline{|\dot{y}(0)|}$ is defined as the relative cross-track speed of same route aircraft pair. No data is available for estimation of this parameter so we take a conservative value of 1 knot as given in the EMA Handbook.

3.5 Estimation of the Parameter Q (k)

Q (k) is defined as the proportion of aircraft pairs with initial longitudinal separation k. We estimate its value from the TSD. Flights entering the FIR on different routes and assigned different flight levels were considered separately (see Figure 5), and the waiting times between successive arrivals were tabulated in minutes. We assume an average speed of 8 NM per minute, and compute the proportion Q (k) as

$$Q(k) = \frac{\text{number of flight pairs with inter-arrival distance } 8k}{\text{total number of flight pairs with at least 80 NM separation}}$$

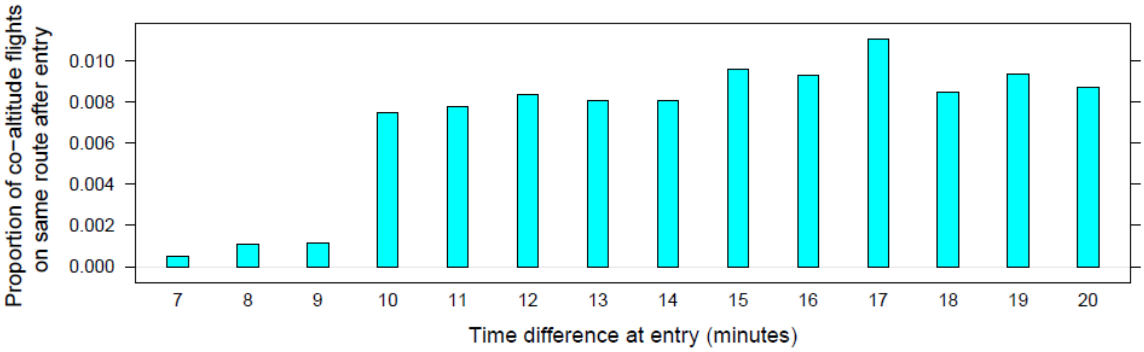

Figure 5: Values of Q(k) estimated from TSD. For co-altitude flights on the same route (after entry / before exit), the proportion of flights that entered k minutes after the preceding flight is plotted for k = 7, 8, 9,. . . , 20 minutes.

The final estimated values of Q (k) for k ranging between 7 and 20 minutes is given in the Table 4.

3.6 Estimation of the Parameter P(K > k)

To estimate $\mathbf{P}$ (K > k) we consider two aircrafts flying on same route at same flight levels at the same direction. Let V and V' be their ground speeds of the front and behind aircraft respectively. We assume these speeds to be statistically independent but identically distributed. Let T_0 be the maximum duration of time before ATC intervenes. Then

$$\mathbf{P}(K > k) = \mathbf{P}\left(0 < \frac{k}{V' - V} < T_0\right) = \mathbf{P}\left(V' - V > \frac{k}{T_0}\right).$$

We note here that the value of T_0 is conservatively taken to be 0.5 hours.

Now we finally estimate these probabilities using the TSD. For that we consider the difference in velocity of two aircraft nominally flying on the same route at the same flight level, after removing records with unusually high or low traversal times. We conservatively consider velocity differences of all flight pairs which are separated by 2 hours time at entry. It is to be noted that we observed from the TSD data that two hours is more than the maximum time taken by any aircraft to travel between its entry and exit points.

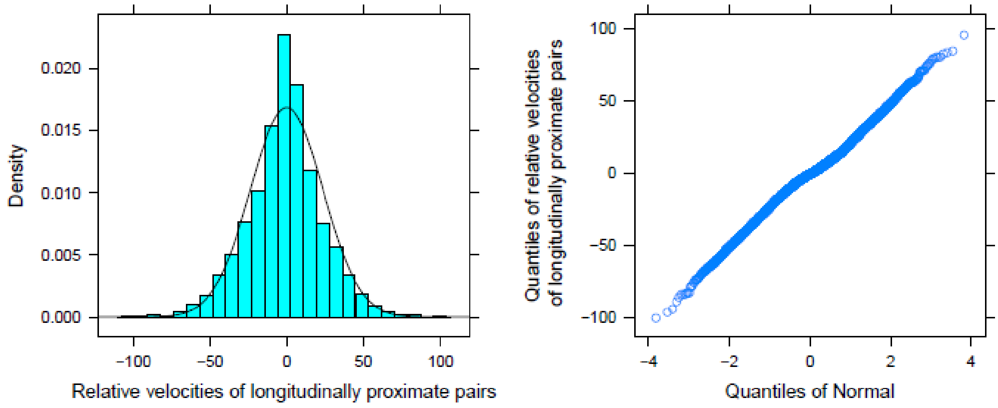

Figure 6: Distribution of relative velocities of longitudinally proximate pairs. The Normal distribution does not necessarily seem to be a reasonable fit.

We observe that these differences in velocity are symmetrically distributed around zero but from the histogram and the quantile- quantile plot (see Figure 6) it is not clear that these differences necessarily Normally distributed. To be conservative, we postulate the following mixture model for the density of these velocity differences.

$$f_v(v) = p\frac{\beta_v}{2}e^{-\beta_v|v|} + (1-p)\frac{1}{\sqrt{2\pi}\sigma_v}e^{-\frac{v^2}{2\sigma_v^2}},$$

which is a mixture of Double Exponential and Normal densities with mixing proportion p. We then estimate the parameters of this mixture model by their maximum likelihood estimates (MLEs). Since this is a mixture model so we use the Expectation-Maximization (EM) algorithm to find the MLEs. The algorithm converged rapidly to give the following estimates:

$$\widehat{p} = 0.3090049,\ \widehat{\beta}_v = 0.1041985,\ \text{ and } \widehat{\sigma}_v = 27.01025\,.$$

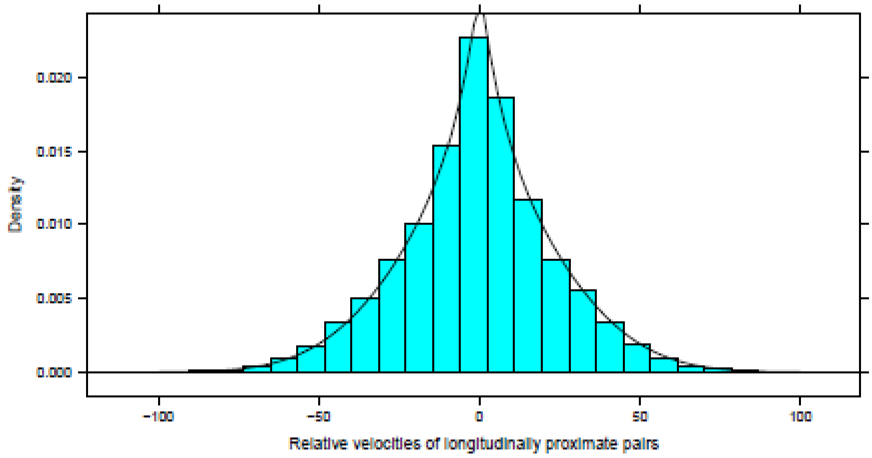

Figure 7: Distribution of relative velocities of laterally proximate pairs along with estimated mixture density (estimated using the EM algorithm).

It is well known in Statistics literature that even though the EM algorithm increases the value of the likelihood it may get trapped in a local maximum. To avoid this problem we tried several starting values and observed that the algorithm always converges to the same estimated values given above.

So it is statistically reasonable to accept the mixing density with these values of the parameters as a good estimate of the true density of the velocity differences. A graphical representation of the fit is given in Figure 7.

With these we estimate the values of P (K > k) for k ranging between 7 and 20. These are presented in the Table 4.

4 Summary of the Safety Assessment

The following table gives a summary of the safety assessment of the BOBASIO region for the month of June 2012.

Type of Risk	Estimated Risk	TLS	Remarks
Lateral Risk	0.740919×10^{-9}	5×10^{-9}	Below TLS
Longitudinal Risk	1.05943×10^{-9}	$5 \times 10^{--9}$	Below TLS

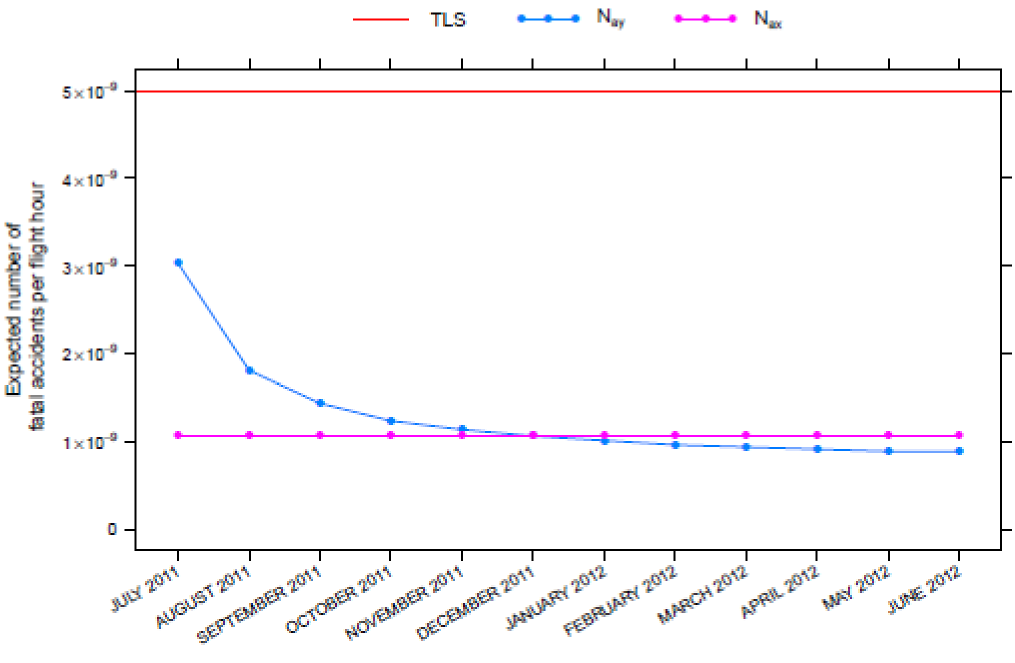

Figure 8: Assessment of Compliance with Lateral and Longitudinal TLS Values.

Figure 8 presents the results of the collision risk estimates for each month using the cumulative 12-month LLD and LLE reports since July 2011.